Notes on degeneracies in the continuum fitting

Vaishali Bhardwaj*

Department of Astronomy, University of Washington, Box 351580, Seattle, WA 98195, USA and
Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720, USA

Andreu Font-Ribera†

Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720, USA and
Institute of Theoretical Physics, University of Zurich,
Winterthurerstrasse 190, 8057 Zurich, Switzerland

Patrick McDonald

Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720, USA

Anže Slosar

Brookhaven National Laboratory, Upton, NY 11973, USA

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Mean flux, mean continuum, quasar amplitudes... everything is degenerated!

I. QUASAR CONTINUUM MODEL

We want to extract information about the Lyα absorption from a set of N quasar lines of sight. We will use $f_q(\lambda)$ to refer to the observed flux in the spectrum of quasar q, as a function of observed wavelength. From now, we won't take into account the fact that we observe a set of finite size pixels, and work with the spectra as if they were continuous functions.

Even though in the real we will take into account things like sky flux, calibration vectors and observational noise, here we keep it simple and model our observed flux as:

$$f_q(\lambda) = C_q(\lambda)\ F_q(\lambda) \tag{1}$$

where $C_q(\lambda)$ is the continuum of the quasar and $F_q(\lambda)$ the Lyα transmission at a given redshift $z = \lambda/\lambda_\alpha - 1$ (with $\lambda_\alpha = 1215.67\mathring{A}$ the restframe wavelength of the Lyα transition) along the line of sight to quasar q.

The Lyα transmission $F_q(\lambda)$ is obviously one for restframe wavelengths larger than the Lyα emission line (we will use the term *red pixels* to refer to these). For smaller restframe wavelengths (from now on, *blue pixels*), its value can be expressed as a perturbation $\delta_q(\lambda)$ around the mean transmission $\bar{F}(\lambda)$ at a particular redshift :

$$F_q(\lambda) = \bar{F}(\lambda)\ [1 + \delta_q(\lambda)]\ . \tag{2}$$

Traditionally, the continuum $C_q(\lambda)$ of high signal to noise spectra has been fitted by eye. The large size of the SDSS quasar survey, as well as the poorer signal to noise of its spectra, required fully automated continuum fits. The approach used here is based in that developed during the first SDSS analysis [1], and in the first BOSS analysis [2].

In short, we assume that the shape of the continuum is the same in all quasars. The continuum is then shifted horizontally as a function of quasar redshift z_q, and vertically as a function of quasar magnitude:

$$C_q(\lambda) = A_q\ \bar{C}(\lambda_{\rm rest})\ , \tag{3}$$

where $\lambda_{\rm rest}$ is the restframe wavelength $\lambda = \lambda_{\rm rest}[1 + z_q]$.

In this analysis we introduce another free parameter per quasar B_q, to control the amplitude inside the forest, defined as a certain restframe range ($\lambda^1_{\rm rest}$,$\lambda^2_{\rm rest}$). For simplicity, in these notes we will not consider blue pixels outside the forest, and assume that everything bluer than the Lyα emission line is part of the forest.

We can now write our model for the continuum in blue pixels (^{B})

$$f^F_q(\lambda) = A_q\ B_q\ \bar{C}(\lambda_{\rm rest})\ \bar{F}(\lambda)\ [1 + \delta_q(\lambda)]\ , \tag{4}$$

* author list alphabetized

†Electronic address: `afont@lbl.gov`

and in red pixels (^{R})

$$f_q^R(\lambda) = A_q \; \bar{C}(\lambda_{\rm rest}) \; . \tag{5}$$

II. DEGENERACIES

In order to fit our set of spectra we need to measure two global functions ($\bar{F}(\lambda)$ and $\bar{C}(\lambda_{\rm rest})$) and two amplitudes for each spectrum (A_q,B_q).

A. Amplitude degeneracies

It is easy to see that these parameters are degenerated, and we can not be able to measure all of them, but only some combination of these. For instance, we could divide all our parameters A_q by a given factor, and multiply $\bar{C}$ by the same factor. Equivalently, we could divide all parameters B_q by a certain factor, and reabsorbe it by multiplying $\bar{F}$ by the same amount.

In order to break these degeneracies, we normalize our mean continuum $\bar{C}$ by requiring its integral over the Lyα forest to be unity, and we fix the value of $\bar{F}$ at a certain redshift $\bar{F}(z = 2.25) = 0.8$.

B. Power law degeneracies

There is a more subtle degeneracy between the slope of the mean transmission $\bar{F}(\lambda)$ and that of the mean continuum $\bar{C}(\lambda_{\rm rest})$, given the relation between observed and restframe wavelength $\lambda = \lambda_{\rm rest}[1 + z_q]$. We can always fit both $\bar{F}$ and the blue part of $\bar{C}$ with a power law, and decompose the functions as a product of the power law and a residual *flat* function:

$$\bar{F}(\lambda) = \lambda^\phi \; \hat{F}(\lambda) \tag{6}$$

and

$$\bar{C}(\lambda_{\rm rest}) = \lambda_{\rm rest}^\gamma \; \hat{C}(\lambda_{\rm rest}) \; . \tag{7}$$

Let's now look at the model for blue pixels of a given quasar:

$$\begin{aligned} f_q^F(\lambda) &= A_q \; B_q \; \lambda_{\rm rest}^\gamma \; \hat{C}(\lambda_{\rm rest}) \; \lambda^\phi \; \hat{F}(\lambda) \; [1 + \delta_q(\lambda)] \\ &= A_q \; B_q \; \left[\frac{\lambda}{1+z_q}\right]^\gamma \; \hat{C}\left(\frac{\lambda}{1+z_q}\right) \; \lambda^\phi \; \hat{F}(\lambda) \; [1 + \delta_q(\lambda)] \\ &= A_q \; \frac{B_q}{[1+z_q]^\gamma} \; \lambda^{\gamma+\phi} \; \hat{C}\left(\frac{\lambda}{1+z_q}\right) \; \hat{F}(\lambda) \; [1 + \delta_q(\lambda)] \; . \end{aligned} \tag{8}$$

It is clear now that we can not constrain the slope of both power laws. In other words, we can tilt $\bar{F}$ and reabsorb this by tilting the blue part of $\bar{C}$ in the oposite direction.

In order to break this degeneracy, we can fix the slope of the best fit power law of one of the two components to an arbitrary value, and tilt the other function accordingly.

[AF: *We should figure out the most efficient / cleanest way to do this.*]

Probably the easiest way to do this is the following:

- Normalize mean flux at $\bar{F}(z = 2.25) = 0.8$.
- Measure a logarithmic slope by computing the following integrals over a certain relevant redshift range ($2.0 < z < 4.0$, for example):

$$\alpha = \frac{\int dz \; \ln\left(\bar{F}(z)/0.8\right)}{\int dz \; \ln\left([1+z]/3.25\right)} \tag{9}$$

- Tilt the mean flux to have the desired logarithmic slope ϕ:

$$\bar{F}(z)' = \bar{F}(z) \left[\frac{1+z}{3.25}\right]^{\phi-\alpha} \tag{10}$$

- Tilt the mean continuum in the opposite direction:

$$\bar{C}(\lambda_{\rm rest})' = \bar{C}(\lambda_{\rm rest})\ \lambda_{\rm rest}^{\alpha-\phi}\ . \tag{11}$$

- Normalize the mean continuum as usual.

[1] P. McDonald, U. Seljak, S. Burles, D. J. Schlegel, D. H. Weinberg, R. Cen, D. Shih, J. Schaye, D. P. Schneider, N. A. Bahcall, et al., ApJS **163**, 80 (2006), arXiv:astro-ph/0405013.
[2] A. Slosar, A. Font-Ribera, M. M. Pieri, J. Rich, J.-M. Le Goff, É. Aubourg, J. Brinkmann, N. Busca, B. Carithers, R. Charlassier, et al., JCAP **9**, 1 (2011), 1104.5244.